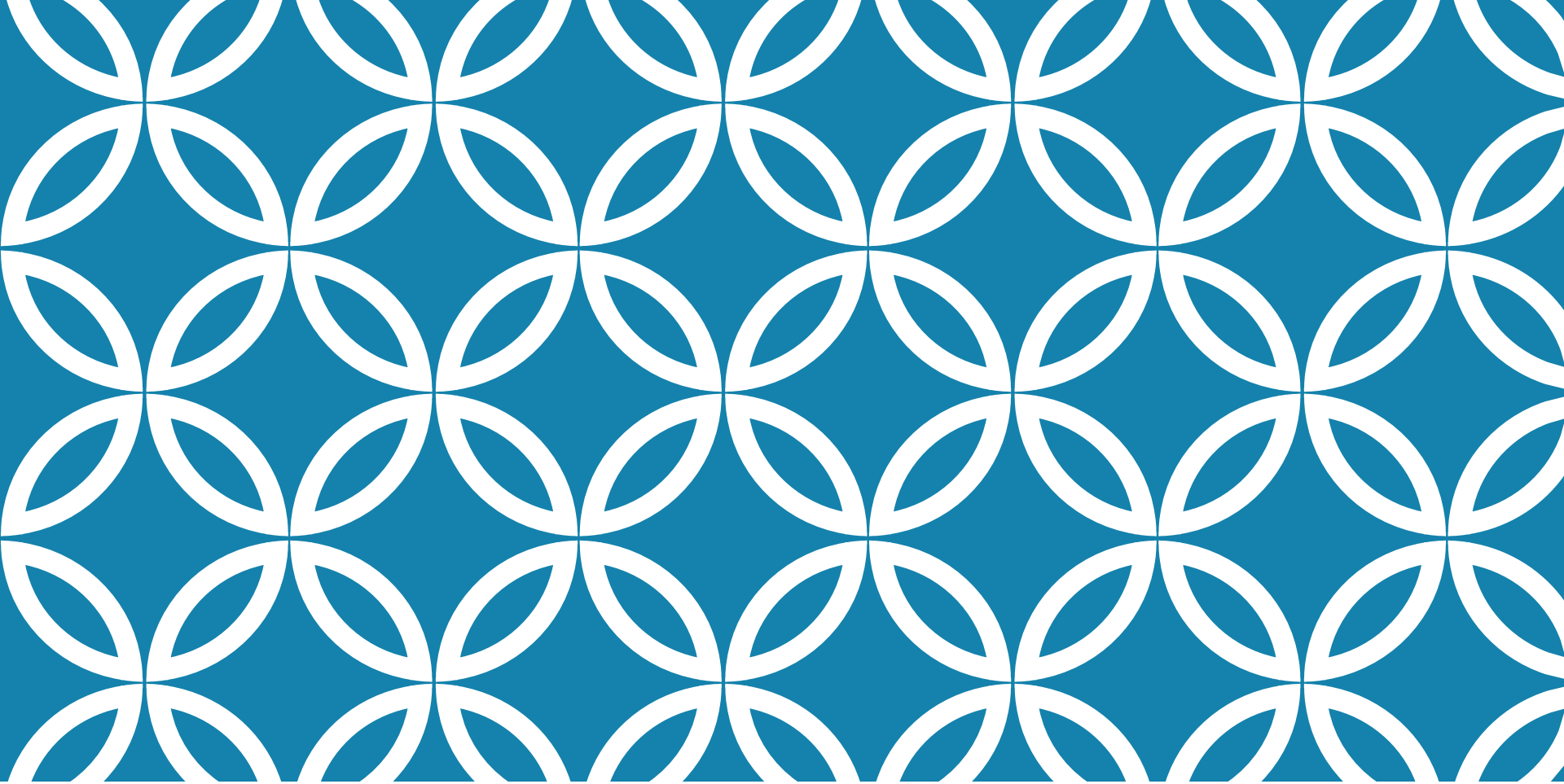


DISCRETE PROBABILITY DISTRIBUTIONS

Chapter 3

WHAT IS A PROBABILITY DISTRIBUTION?

PROBABILITY DISTRIBUTION A listing of all the outcomes of an experiment and the probability associated with each outcome.

CHARACTERISTICS OF A PROBABILITY DISTRIBUTION

1. The probability of a particular outcome is between 0 and 1 inclusive.
2. The outcomes are mutually exclusive.
3. The list of outcomes is exhaustive. So the sum of the probabilities of the outcomes is equal to 1.

PROBABILITY DISTRIBUTION EXAMPLE

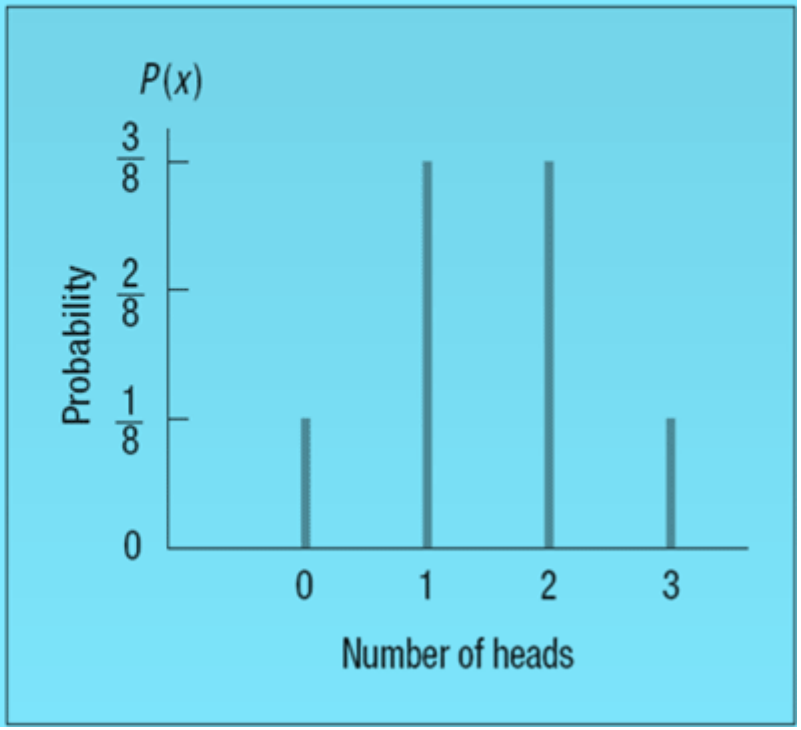
Suppose we are interested in the number of heads showing face up with 3 tosses of a coin

The possible outcomes are 0 heads, 1 head, 2 heads, and 3 heads

Possible Result	Coin Toss			Number of Heads
	First	Second	Third	
1	T	T	T	0
2	T	T	H	1
3	T	H	T	1
4	T	H	H	2
5	H	T	T	1
6	H	T	H	2
7	H	H	T	2
8	H	H	H	3

PROBABILITY DISTRIBUTION TABLE

Number of Heads, x	Probability of Outcome, $P(x)$
0	$\frac{1}{8} = .125$
1	$\frac{3}{8} = .375$
2	$\frac{3}{8} = .375$
3	$\frac{1}{8} = .125$
Total	$\frac{8}{8} = 1.000$



RANDOM VARIABLES

In any experiment of chance, the outcomes occur randomly, and so are called random variables

RANDOM VARIABLE is a numerical description of the outcome of an experiment , can assume different values.

Examples

The number of employees absent from the day shift on Monday, the number might be 0, 1, 2, 3, …The number absent is the random variable

TWO TYPES OF RANDOM VARIABLES

One type of random variable is the discrete random variable

DISCRETE RANDOM VARIABLE is a variable have numeric values that can be listed and often can be counted.

Examples

Tossing a coin three times and counting the number of heads

A bank counting the number of credit cards carried by a group of customers

CONTINUOUS RANDOM VARIABLES

CONTINUOUS RANDOM VARIABLES: Is a variable can assume an infinite number of values within a given range.

Example : the (exact) weight of a person is a continuous random variable. Foot length is also a continuous random variable.

Continuous random variables are measured with interval or ratio scale and the likelihood of a continuous random variable can be summarized with a probability distribution.

MEAN AND VARIANCE OF A PROBABILITY DISTRIBUTION

- The mean of a discrete random variable is a weighted average of all the possible values of the random variable X, where each value is weighted by its P(x). probability.

MEAN OF A PROBABILITY DISTRIBUTION $\mu = \Sigma[xP(x)]$ or E(x)

Another term used to describe the mean is expected value.

- Here is the formula for the standard deviation of a discrete random variable. $P(x)$ represents the probability of x, where x is a value of the random variable X, and μ_x is the mean of X.

VARIANCE OF A PROBABILITY DISTRIBUTION $\sigma^2 = \Sigma[(x - \mu)^2 P(x)]$

STANDARD DEVIATION OF A PROBABILITY DISTRIBUTION

Another formula for the variance is:

$$\text{Or Var}(x) = m_2 - (m_1)^2 \qquad \text{Where: } m_2 = \sum x^2 * P(x), \qquad \text{and } m_1 = E(X)$$

The Standard deviation for a Discrete Random Variable is the square roots of the variance.

- $$\sigma = \sqrt{\sigma^2} = \sqrt{\sum (x - E(x))^2 P(x_i)}$$

EXAMPLE

consider the experiment of rolling a die and observing the number on the upward face.

a. define the random variable that represents the number of dots on the upward face? And whether the random variable is discrete or continuous?

b. List the values that the random variable may assume?

c. Assign the frequency and the probability of each possible outcome?

EXAMPLE (CONT.)

d. Is this a valid probability distribution? Explain?
e. What is the probability of $x=1$, $x \leq 3$, $x > 5$.
f. What is the values of E(x), Var(x), and $\sigma(x)$

SOLUTION

a. Let x = number of dots obtained on one roll of die. It is a discrete random variable.

b. It must be $x \in \{1, 2, 3, 4, 5, 6\}$

Number of X obtained	Frequency	Probability P(x)
1	1	1/6
2	1	1/6
3	1	1/6
4	1	1/6
5	1	1/6
6	1	1/6
Total	6	6/6 = 1

SOLUTION (CONT.)

d. This is a valid discrete probability distribution since:
Each probability is between 0 and 1, inclusive, $0 \leq x \leq 1$. And The sum of all probabilities equals 1, $\sum P(x) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$

e. compute the probabilities

p(x=1) = $\frac{1}{6}$

P(x≤3) = $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$ =0,5

p(x>5) = $\frac{1}{6}$

SOLUTION (CONT.)

METRICS		E(x)	Var(x)
x	P(x)	X*p(x)	$(x-E(x))^2$ *p(x)
1	0,17	0,17	1,042
2	0,17	0,333	0,375
3	0,17	0,5	0,042
4	0,17	0,667	0,042
5	0,17	0,833	0,375
6	0,17	1	1,042
		3,,486≈3,5	2,917

$\sigma(x)=\sqrt{Var(x)} =\sqrt{2.917}=1.708$ values of the random variable deviate from E(x) by 1.708

CONTINUOUS PROBABILITY DISTRIBUTION

The probability distribution of a continuous random variable is represented by a probability density curve.

The probability that X has a value in any interval of interest is the area above this interval and below the density curve.

