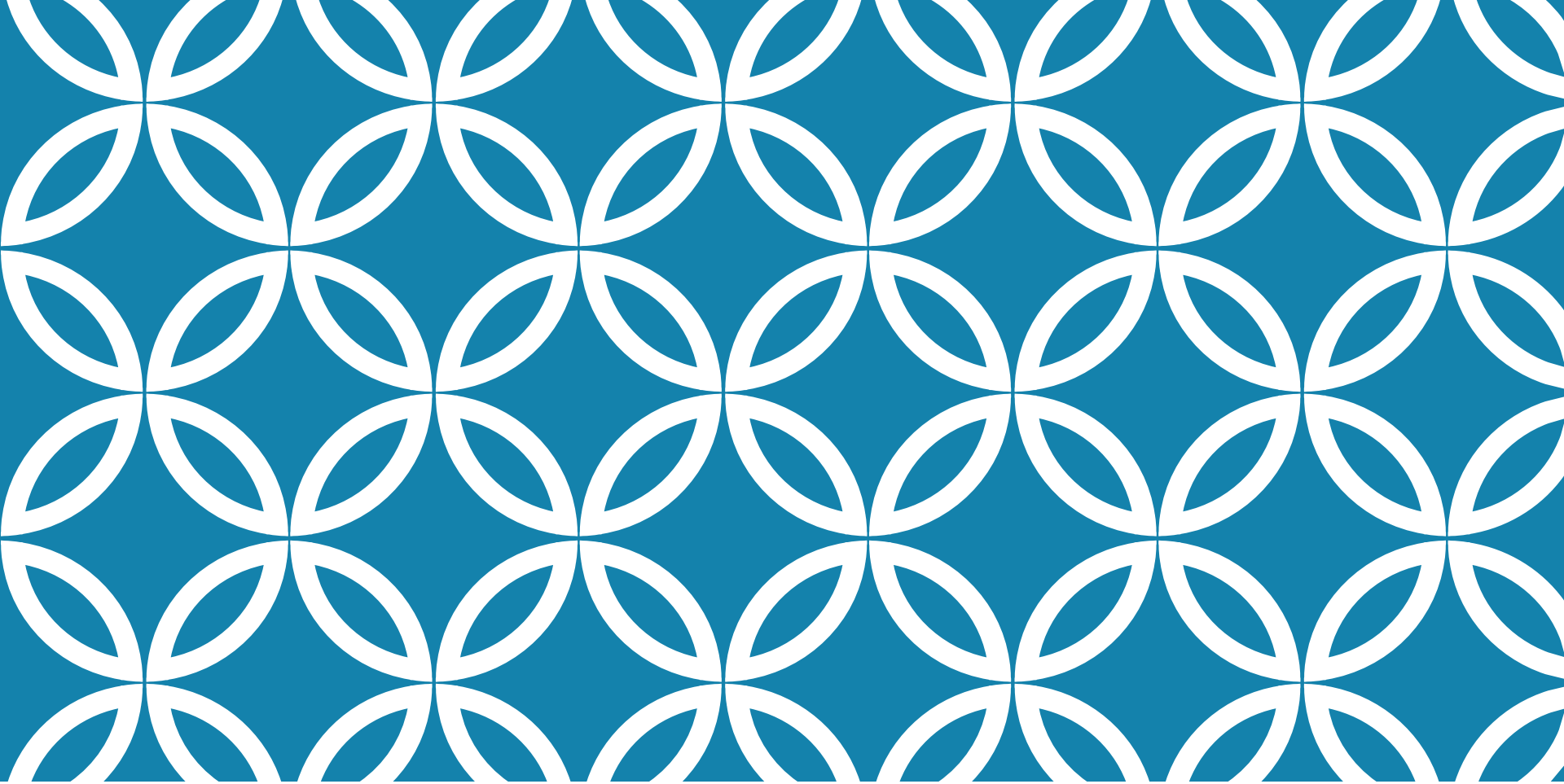


DISCRETE PROBABILITY DISTRIBUTIONS (CONT.)

Chapter 4

BIVARIATE DISTRIBUTION, COVARIANCE AND FINANCIAL PORTFOLIOS

A probability distribution involving two random variables is called a Bivariate Probability Distribution. With bivariate probability distributions, we often want to know the relationship between the two random variables.

The covariance and/or correlation coefficient are good measures.

THE RULES FOR THE BIVARIATE DISTRIBUTION

Covariance of random variables x and y

$$\sigma_{xy} = \frac{[\mathrm{Var}(x+y) - \mathrm{Var}(x) - \mathrm{Var}(y)]}{2} \quad \text{.....[4-1]}$$

Correlation between random variables x and y

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \text{.....[4-2]}$$

Expected value of a linear combination of random variables x and y

$$E(ax + by) = aE(x) + bE(y) \quad \text{..... [4-3]}$$

Variance of linear combination of two random variables

$$\mathrm{Var}(ax + by) = a^2\mathrm{Var}(x) + b^2\mathrm{Var}(y) + 2ab\sigma_{xy} \quad \text{..... [4-4]}$$

Where σ_{xy} is the covariance of x and y

NOTE

the covariance between tow variables measures the degree to which the variables move together relative to their E(x) over time.

A positive covariance means the variables tend to move in the same direction over the sample period.

A negative covariance means the two variables tend to move in the opposite direction

EXAMPLE

Given below is a bivariate distribution for the random variables x and y.

P(x,y)	x	y
0,15	60	70
0,55	20	60
0,3	40	60

- Compute: E(x), E(y), Var(x), Var(y)?
- Develop a probability distribution for (x+y)?
- Compute the covariance and correlation for x and y? explain the relationship between x and y.
- What is the expected value of the linear combination of x and y if the coefficient a= 0.4 and b =0.6?

SOLUTION

a.Compute: E(x), E(y), Var(x), Var(y).

Metrics			E(x)	E(y)	Var(x)	Var(y)
P(x,y)	x	y	x.P(x)	y.P(y)	$(x - E(x))^2 . P(x)$	$(y - E(y))^2 . P(y)$
0.15	60	70	9	10.5	117.6	10.8375
0.55	20	60	11	33	79.2	1.2375
0.3	40	60	12	18	19.2	0.675
			32	61.5	216	12.75

$$\sigma_x = \sqrt{216} = 14.697$$

$$\sigma_y = \sqrt{12.75} = 3.57$$

SOLUTION (CONT.)

b. Developing A Bivariate Probability Distribution

Metrics		E(S)	Var(x+y)= $(S-E(S))^2*P(x,y)$
P(x,y)	(x+y)=S	S*P(x,y)	
0.15	130	19.5	199.8375
0.55	80	44	100.2375
0.3	100	30	12.675
$\sum(p, x) = 1$		93.5	312.75

SOLUTION (CONT.)

c. The covariance: $\sigma_{xy} = \frac{[\mathbf{Var(x+y)} - \mathbf{Var(x)} - \mathbf{Var(y)}]}{2}$

$\boldsymbol{\sigma_{xy}} = \frac{312{,}75 - 216 - 12.75}{2} = 42$

A covariance of 41.875 indicates that there is a positive relationship between x and y.

Correlation: $\boldsymbol{\rho_{xy}} = \frac{\boldsymbol{\sigma_{xy}}}{\boldsymbol{\sigma_x \sigma_y}} = \frac{42}{14.697 * 3.57} = 0{,}8$

this indicates a strong relationship between x and y.

SOLUTION (CONT.)

Note: the correlation measures the linear association between variables.

- ρ_{xy} colse to +1 strong positive linear relationship.
- ρ_{xy} colse to -1 strong negative linear relationship.
- ρ_{xy} colse to 0 There Is no linear relationship.

d. the expected value of the linear combination of x and y if the coefficient a= 0.4 and b =0.6

$E(ax + by) = aE(x) + bE(y)$= (0,4*32)+(0,6*61,5)=49,7

The expected value for the two variables (linear combination (x,y) is 49.7 with 0.4 and 0.6 coefficient respectively.

BINOMIAL DISTRIBUTION

There are four requirements of a binomial probability distribution

1. There are only two possible outcomes and the outcomes are mutually exclusive, as either a success (S) or a failure (F)
2. The number of trials is fixed and known
3. The probability of a success is the same for each trial
4. Each trial is independent of any other trial

Example

A young family has two children, both boys. The probability of the third birth being a boy is still .50. The gender of the third child is independent of the gender of the other two.

BINOMIAL PROBABILITY EXPERIMENT

Use the number of trials, n, and the probability of a success, P, to compute binomial probability

BINOMIAL PROBABILITY EXPERIMENT

1. An outcome on each trial of an experiment is classified into one of two mutually exclusive categories — a success or a failure.
2. The random variable is the number of successes in a fixed number of trials.
3. The probability of success is the same for each trial.
4. The trials are independent, meaning that the outcome of one trial does not affect the outcome of any other trial.

HOW IS A BINOMIAL PROBABILITY COMPUTED?

Binomial Probability Function:

$$P(x) = C_n^x p^x (1-p)^{(n-x)} \ldots\ldots\ldots [4\text{-}5].$$

Where:

X= the number of successes

P=the probability of successes in one trial

n=the number of trials

P(x) = the probability of successes X in (n) trials

$$C_n^x = \frac{n!}{x!\,(n-x)!}$$

EXAMPLE 1

let us consider the experiment of tossing a coin 6 times n=6. if the random variable of interest is x= heads we think of it as success, then p=0.5.

does this experiment satisfy the binomial experiment conditions?

Solution

- n =6 the experiment consists of 6 identical trials.
- Two possible outcomes for each trial (head or tail).
- The probability of success (head) and failure (tail) are the same for each trial p=0.5 and 1-p=0.5
- The trials are independent because the outcome on any one trial is not affected by what happens on the other trials.

Thus, the properties of a binomial experiment are satisfied, and the random variable x can assume the values x= 0,1,2,3,4,5,6.

SHORTCUT FORMULAS

Expected value and variance for the binomial distribution

- The mean or the expected value **μ** = **E(x)** = n*P…….[4-6]
- The variance **Var(x)** =∂^2 = n*P*(1*-P) ……..[4-7]

The standard deviatoin Is the square root of the variance So

$\partial = \sqrt{n*P*(1-P)}$……[4-8]

EXAMPLE 2

A survey found that 17% of U.S. adults say that Google News is a major source of news for them. You randomly select four adults and ask them whether Google News is a major source of news for them.

› Find the probability that (A) exactly two of them respond yes, (B) at least two of them respond yes, and (C) fewer than two of them respond yes. Find the expected number and the variance of adults who will say yes to the statement,

› Solution :

a. Using $n = 4$, $p = 0.17$, $q = 0.83$, and $x = 2$, the probability that exactly two adults will respond yes is : $P(2) = {}_4C_2(0.17)^2(0.83)^2 = 6(0.17)^2(0.83)^2 \approx 0.119$.

b. To find the probability that at least two adults will respond yes, find the sum of $P(2)$, $P(3)$, and $P(4)$. Begin by using the binomial probability formula to write an expression for each probability.

$P(2) = {}_4C_2(0.17)^2(0.83)^2 = 6(0.17)^2(0.83)^2$

$P(3) = {}_4C_3(0.17)^3(0.83)^1 = 4(0.17)^3(0.83)^1$

$P(4) = {}_4C_4(0.17)^4(0.83)^0 = 1(0.17)^4(0.83)^0$

EXAMPLE 2 (CONT.)

So, the probability that at least two will respond yes is

$$P(x \geq 2) = P(2) + P(3) + P(4) = 6(0.17)^2(0.83)^2 + 4(0.17)^3(0.83)^1 + 1(0.17)^4(0.83)^0 \approx 0.137.$$

c. To find the probability that fewer than two adults will respond yes, find the sum of $P(0)$ and $P(1)$.

$$P(0) = {}_4C_0(0.17)^0(0.83)^4 = 1(0.17)^0(0.83)^4$$
$$P(1) = {}_4C_1(0.17)^1(0.83)^3 = 4(0.17)^1(0.83)^3$$

So, the probability that fewer than two will respond yes is

$$P(x < 2) = P(0) + P(1) = 1(0.17)^0(0.83)^4 + 4(0.17)^1(0.83)^3 \approx 0.863.$$

The expected number and the variance of adults who will say yes to the statement, 'Google News is a major source of news for us.

E(x)= n*p = 4*0,17 = 0,68

V(x)=n*p*q=4*0,17*(1-0,17)=0,5644

CUMULATIVE BINOMIAL PROBABILITY DISTRIBUTIONS

A study by the Illinois Department of Transportation concluded that 76.2% of front seat occupants wore seat belts. That is, both occupants of the front seat were using their seat belts. Suppose we decide to compare that information with current usage. We select a sample of 12 vehicles.

1. What is the probability that the front seat occupants in exactly 7 of the 12 vehicles are wearing seat belts?

$$P(x) = {}_nC_r(p)^r(1-p)^{n-r}$$

$$P(x=7) = {}_{12}C_7(.762)^7(1-.762)^{12-7} = 792(.149171)(.000764) = .0902$$

2. What is the probability that at least 7 of the 12 front seat occupants are wearing seat belts?

$$\begin{aligned} P(x\geq 7) &= P(x=7) + P(x=8) + P(x=9) + P(x=10) + P(x=11) + P(x=12) \\ &= .0902 + .1805 + .2569 + .2467 + .1436 + .0383 \\ &= .9562 \end{aligned}$$