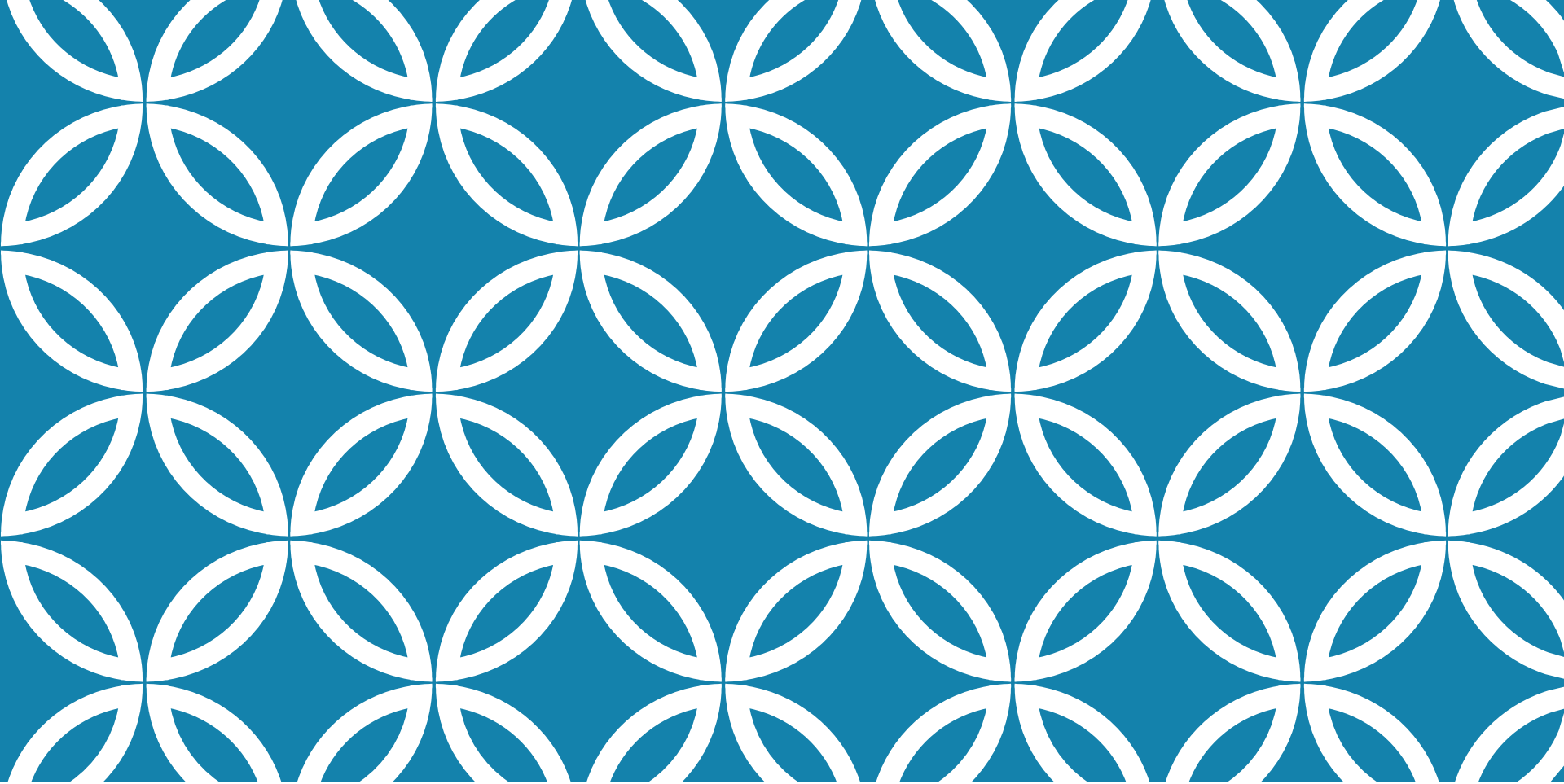


POISSON PROBABILITY DISTRIBUTION

Chapter 6

6-1

STATISTICS 03

POISSON PROBABILITY DISTRIBUTION

In this lecture we consider a discrete random variable that is often useful in estimating the number of occurrences over a specified interval of time or space.

This describes the number of times some event occurs during a specified interval

The interval can be time, distance, area, or volume

This distribution based on Two assumptions

- The probability is proportional to the length of the interval
- The intervals are independent

POISSON PROBABILITY DISTRIBUTION

For example, the random variable of interest might be the number of arrivals at a car wash in one hour, the number of repairs needed in 10 miles of highway, or the number of leaks in 100 miles of pipeline.

POISSON PROBABILITY EXPERIMENT

1. The random variable is the number of times some event occurs during a defined interval.
2. The probability of the event is proportional to the size of the interval.
3. The intervals do not overlap and are independent.

THE POISSON PROBABILITY DISTRIBUTION FUNCTION

The Poisson probability distribution function is defined by the following equation:

$$P(x) = \frac{(\lambda)^x e^{-\lambda}}{x!} \quad \text{.....[1-5]}$$

Where:

P(x)= the probability of x occurrences in an interval.

$\lambda\lambda$ = expected value or mean number of occurrences in an interval (μ or $E(x)$

ee= 2.71828

NOTE

for the Poisson probability distribution, x is a discrete random variable indicating the number of occurrences in the interval. Since there is no stated upper limit for the number of occurrences, the probability function p(x) is applicable for values x=0,1,2, …without limit.

In practical application, x will eventually become large enough so that f(x) is approximately to zero and the probability of any larger values of x becomes negligible.

EXAMPLE 01

The mean (average) number of accidents per month at a certain intersection is 3. What is the probability that in any given month 4 accidents will occur at this intersection?

Solution:

Firstly here we have a Poisson distribution with $\lambda=3$

And X=4

$$P(x=4) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{3^4 e^{-3}}{4!} = 0.168$$

In any given month there is a probability of 0.168 that 4 accidents will occur at this intersection.

EXAMPLE 02

Suppose that we are interested in the number of patients who arrive at the emergency room of a hospital during a 15-minute period on weekday mornings. If we can assume that the probability of a patient arriving is the same for any two time periods of the arrival or nonarrival in any other time period. The Poisson probability function is applicable.

Let us assume that the historical data shows that the average number of patients arriving in a 15-minute period of time is 10. What is the probability of exactly 5 arrivals in 15 minutes.

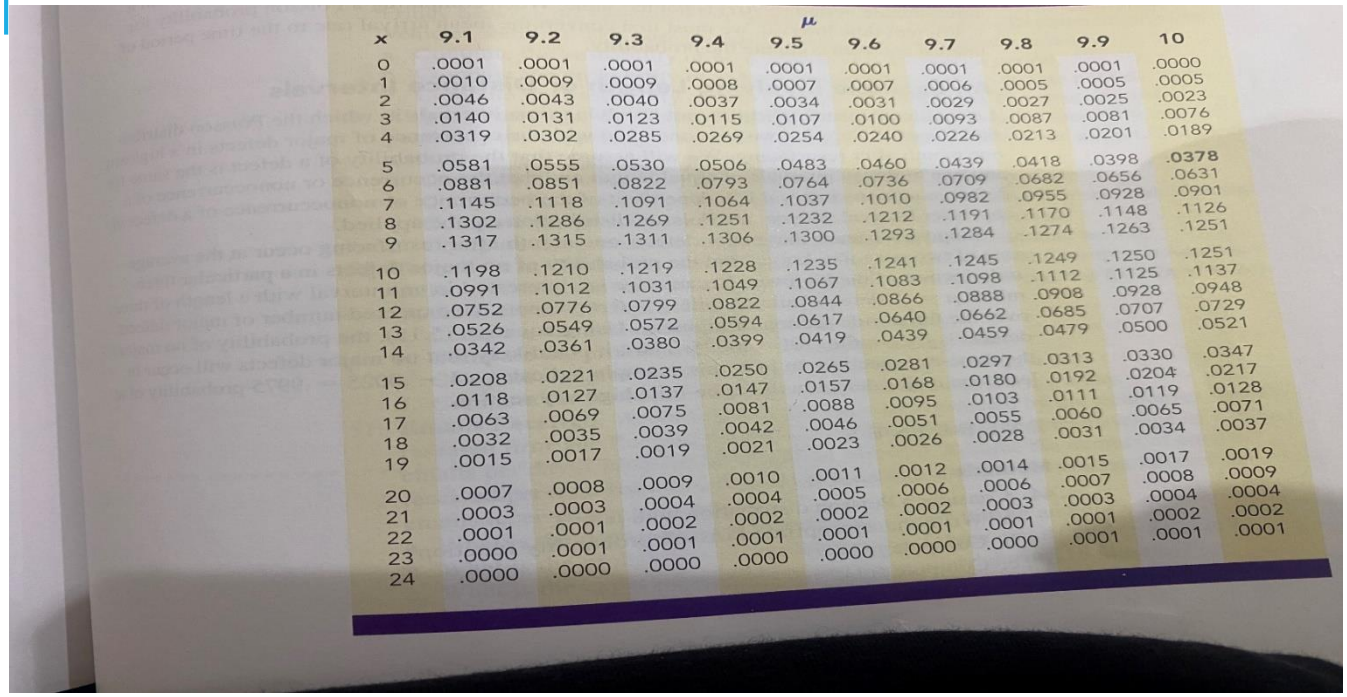
EXEMPLE 02 (CONT.)

Solution

$$P(x=5) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{10^5 e^{-10}}{5!} = 0{,}038$$

Although this probability was determined by evaluating the probability function with λ=10 and x=5, it is often easier to refer to a table for the Poisson distribution. The table provides probabilities for specific values of x and λ or μ. To use the table we need to know only x and λ.

POISSON DISTRIBUTION TABLE



x	μ = 9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10
0	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0000
1	.0010	.0009	.0009	.0008	.0007	.0007	.0006	.0005	.0005	.0005
2	.0046	.0043	.0040	.0037	.0034	.0031	.0029	.0027	.0025	.0023
3	.0140	.0131	.0123	.0115	.0107	.0100	.0093	.0087	.0081	.0076
4	.0319	.0302	.0285	.0269	.0254	.0240	.0226	.0213	.0201	.0189
5	.0581	.0555	.0530	.0506	.0483	.0460	.0439	.0418	.0398	.0378
6	.0881	.0851	.0822	.0793	.0764	.0736	.0709	.0682	.0656	.0631
7	.1145	.1118	.1091	.1064	.1037	.1010	.0982	.0955	.0928	.0901
8	.1302	.1286	.1269	.1251	.1232	.1212	.1191	.1170	.1148	.1126
9	.1317	.1315	.1311	.1306	.1300	.1293	.1284	.1274	.1263	.1251
10	.1198	.1210	.1219	.1228	.1235	.1241	.1245	.1249	.1250	.1251
11	.0991	.1012	.1031	.1049	.1067	.1083	.1098	.1112	.1125	.1137
12	.0752	.0776	.0799	.0822	.0844	.0866	.0888	.0908	.0928	.0948
13	.0526	.0549	.0572	.0594	.0617	.0640	.0662	.0685	.0707	.0729
14	.0342	.0361	.0380	.0399	.0419	.0439	.0459	.0479	.0500	.0521
15	.0208	.0221	.0235	.0250	.0265	.0281	.0297	.0313	.0330	.0347
16	.0118	.0127	.0137	.0147	.0157	.0168	.0180	.0192	.0204	.0217
17	.0063	.0069	.0075	.0081	.0088	.0095	.0103	.0111	.0119	.0128
18	.0032	.0035	.0039	.0042	.0046	.0051	.0055	.0060	.0065	.0071
19	.0015	.0017	.0019	.0021	.0023	.0026	.0028	.0031	.0034	.0037
20	.0007	.0008	.0009	.0010	.0011	.0012	.0014	.0015	.0017	.0019
21	.0003	.0003	.0004	.0004	.0005	.0006	.0006	.0007	.0008	.0009
22	.0001	.0001	.0002	.0002	.0002	.0002	.0003	.0003	.0004	.0004
23	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002
24	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001

THE EXPECTED VALUE AND VARIANCE FOR POISSON PROBABILITY DISTRIBUTION:

A property of the Poisson distribution is that the expected value E(x)= λ of the distribution and the variance of the distribution are equal.

So E(x) = V(X) = λ

In Our precedent example the expected value of the number of arrivals during 15-minute period is $E(x)= \lambda =10$

the variance for the number of arrivals during 15-minute period is V(x)= 10

The standard deviation is $\sigma=\sqrt{10}=3.16$.

THE EXPECTED VALUE AND VARIANCE FOR POISSON PROBABILITY DISTRIBUTION:(CONT.)

Our example involves a 15-minute period, but other time periods can be used.

EXAMPLE 02

Suppose we want to compute the probability of 1-arrival in a 3- minute period. Because 10 is the expected number of arrivals in a 15-minute period, we see that $10/15=2/3$ is the expected number of arrivals in a 1-minute period and that $(2/3)$*(3 minutes) $= 2$ is the expected number of arrivals in a 3-minute period.

Thus, the probability of x arrivals in a 3-minute time period with $\lambda=2$ is given by the following Poisson probability function:

$$P(X) = \frac{2^{x}e^{-2}}{x!}$$

EXAMPLE 02 (CONT.)

The probability of 1 arrival in a 3-minute period is calculated as follows:

$$P(x=1) = \frac{2^1 e^{-2}}{1!} = 0{,}2707$$

One might expect that because (5 arrivals)/5=1 arrival and (15 minutes)/5= 3 minutes, we would get the same probability for one arrival during a 3-minute period as we do for 5 arrivals during a 15-minute period.

Earlier we calculated the probability of 5 arrivals in a 15-minute period to be 0.0378. However, note that the probability of 1 arrival in a 3-minute period (0.2707) is not the same.

When computing a Poisson probability for a different time interval, we must first convert the λ or mean arrival rate to the time period of interest and then compute the probability.

EXAMPLE 03

According to the National Atmospheric administration, the state of Florida averages 18 tornadoes every September (there are 30 days September).

a. Compute the mean number of tornadoes per day?

b. Compute the probability of no tornadoes during a day?

c. Compute the probability of more than 1 tornado during a day?

Solution:

a. The mean number of tornadoes per day = 18/30=0.6

EXAMPLE 02 (CONT.)

b. $P(x=0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{0.6^0 e^{-0.6}}{0!} = 0{,}549$

c. $P(x>1) = 1 - p(x \leq 1) = 1 - (p(x=0) + p(x=1))$

$1 - (0{,}549 + \frac{0.6^1 e^{-0.6}}{1!}) = 1 - (0{,}549 + 0{,}329) = 0{,}112$